Introduction to Accelerated Magnetic Resonance Imaging

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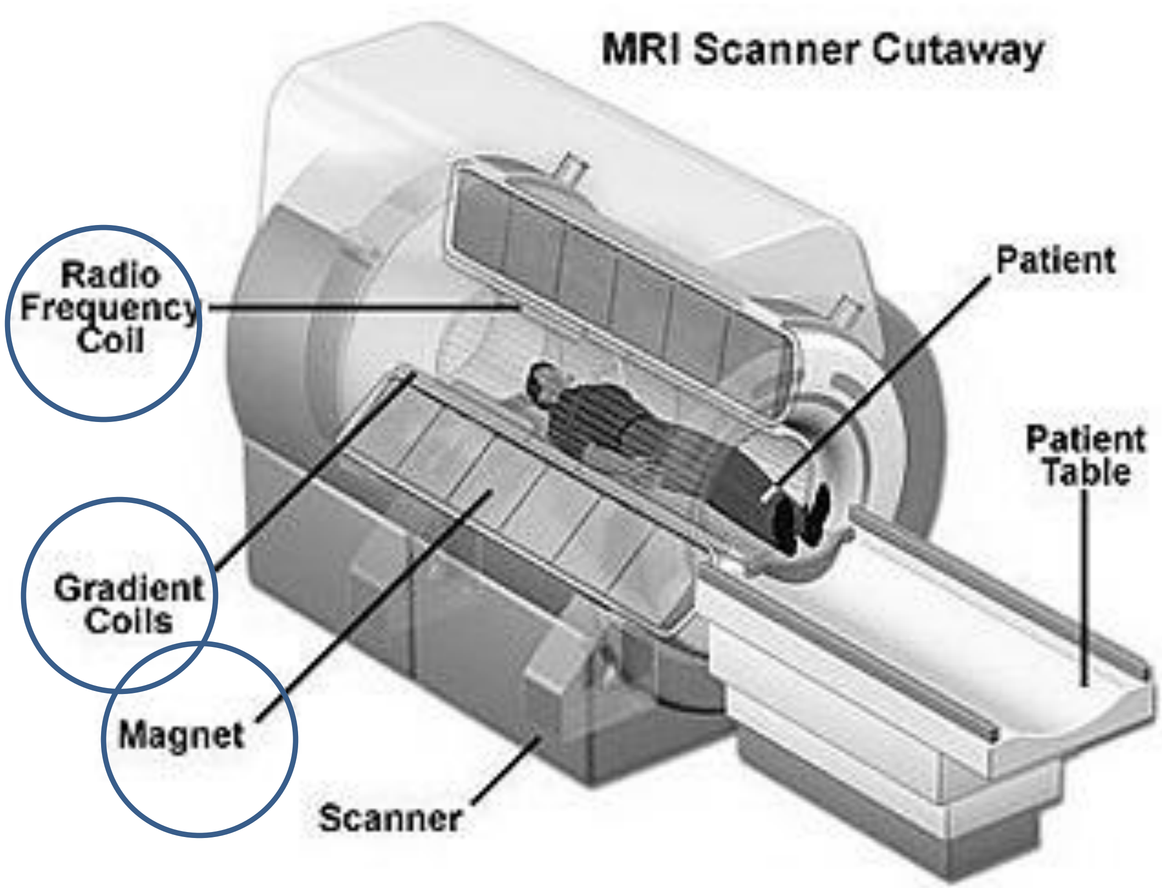
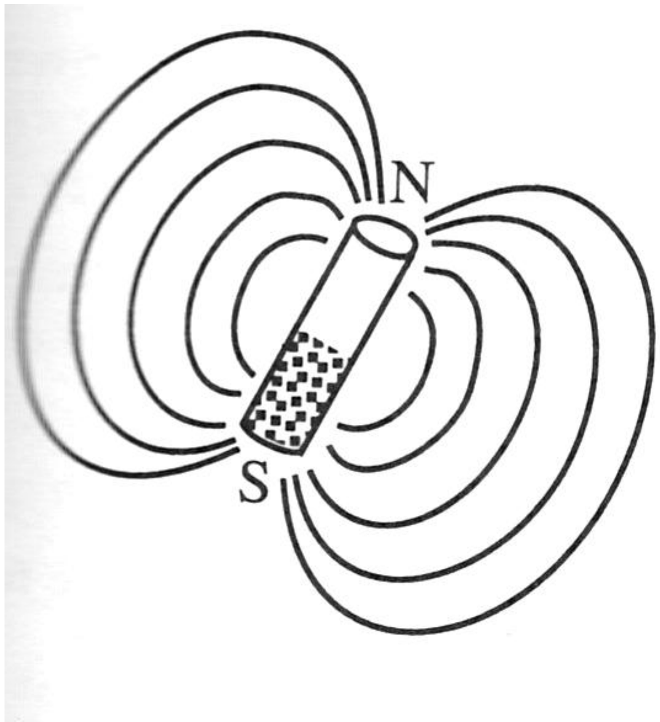
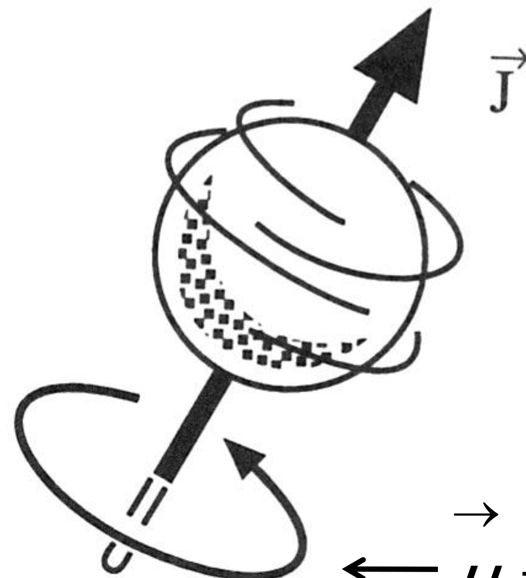
A typical MRI Scanner



The origin of MRI signal





$$\longleftarrow \vec{\mu} = \gamma \vec{J}$$

γ- Gyromagnetic ratio
μ-magnetic moment

- Nuclei with odd atomic weights/numbers possess *spin(J)*
- *Nuclear magnetism: Spin system in a magnetic field*

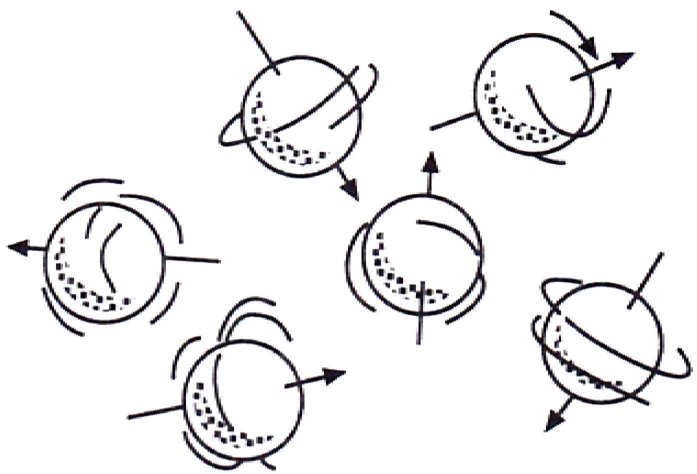
-Proton has electrical charges

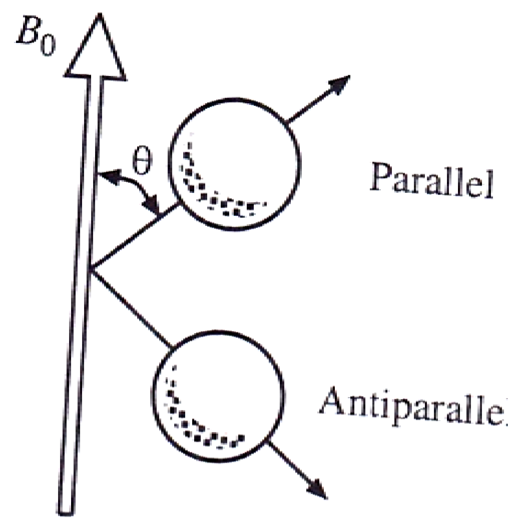
-Rotates about its own axis if it has non-zero spin

Nuclear magnetic moments (μ)

- A vector whose magnitude is given by $\gamma = \sqrt{h(h+1)}$

  -*h*-Planck's constant
- The direction of μ is determined by the main magnetic field (B_0)





$$\vec{M} \propto \left(N_{\uparrow} - N_{\downarrow}\right) \approx N_s \frac{\lambda h B_0}{2KT_s}$$

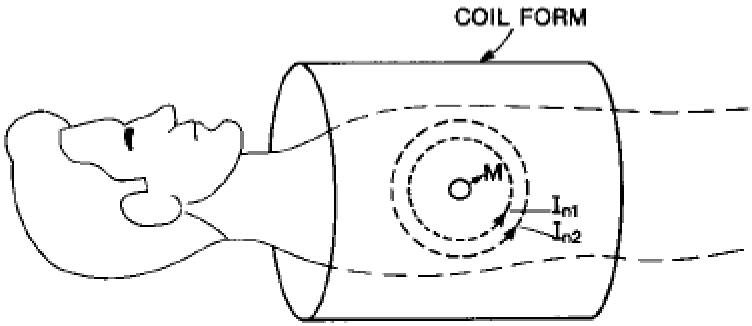
Role of hardware in Imaging

Hardware	Imaging parameter
The magnet	Signal to Noise (SNR)
Gradient coils	Fourier data acquisition
RF Antenna	Role in data acquisition

- Magnet-Intrinsic SNR- Indispensable to sparse signal recovery
- Gradient coils- k-space acquisition-Limits acquisition speed
- RF Antennas- Important role in sparse signal recovery

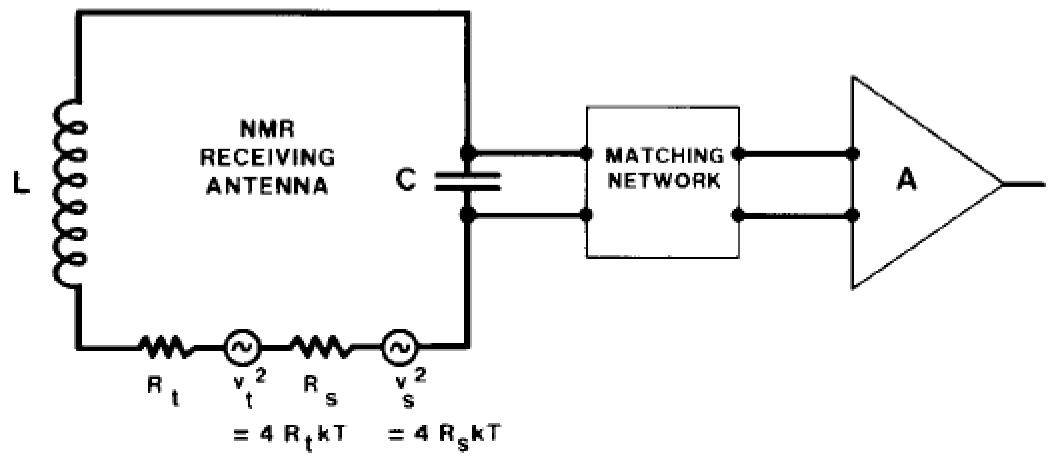
Intrinsic SNR

- Intrinsic SNR is the ratio of the signal from the magnetization M to the signals produced by thermally generated noise currents



$$\vec{M} \propto B_0$$

Intrinsic SNR



- k - Boltzmann constant
- T- Absolute temperature in degree Kelvin

$S \propto \frac{dM}{dt} \propto \omega M$ and $\omega = \gamma B_0, M \propto B_0$

Intrinsic SNR

Therefore, $$S \propto B_0^2$$

But we also know[1], $$v_s \propto B_0, v_t \propto \sqrt{B_0}$$

Therefore at high field strengths, Intrinsic SNR $$\frac{S}{v_s} \propto B_0$$

1. D. I. HOULTA ND P. C. LAUTERBUR *J., Magn. Reson.* ***34,425 (1 979).***

The objective of Accelerated MRI is to minimize data acquisition time..

The signal acquisition model

$$s(t,t_y) = \iint S_l(x,y) M(x,y) e^{-i\gamma G_y y t_y} e^{-i\gamma G_x x t} dxdy$$

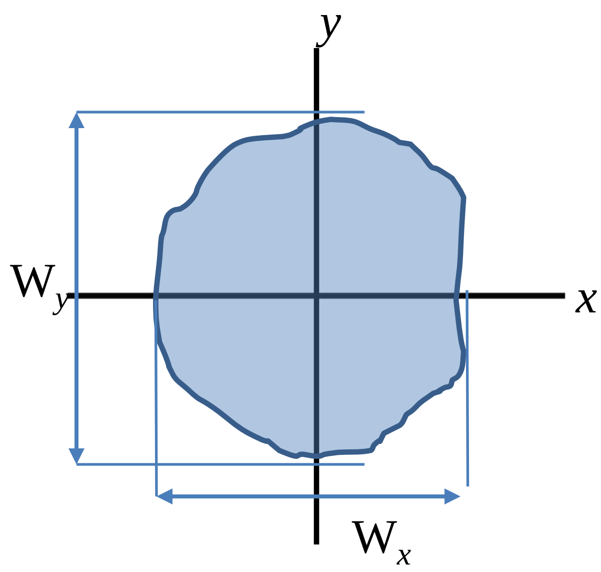
- M – Magnetization distribution in x-y plane
- t_y – phase encoding gradient on-time
- t – Frequency encoding gradient on time
- G_y- phase encoding gradient amplitude
- G_x- Frequency encoding gradient amplitude
- S_l - Signal weighting by the l^{th} RF antenna

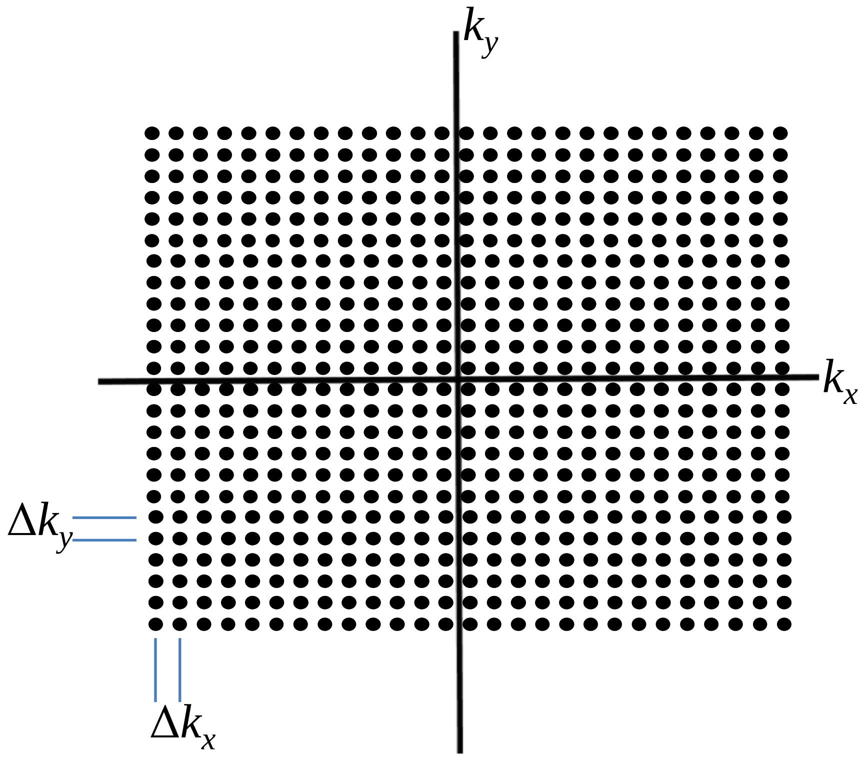
$$F(k_x,k_y) = \iint S_l(x,y) M(x,y) e^{-i2\pi k_y y} e^{-i2\pi k_x x} dxdy$$

$$k_x = \left(\frac{\gamma}{2\pi}\right) G_x t$$

$$k_x = \left(\frac{\gamma}{2\pi}\right) G_y t_y$$

Signal sampling requirements..

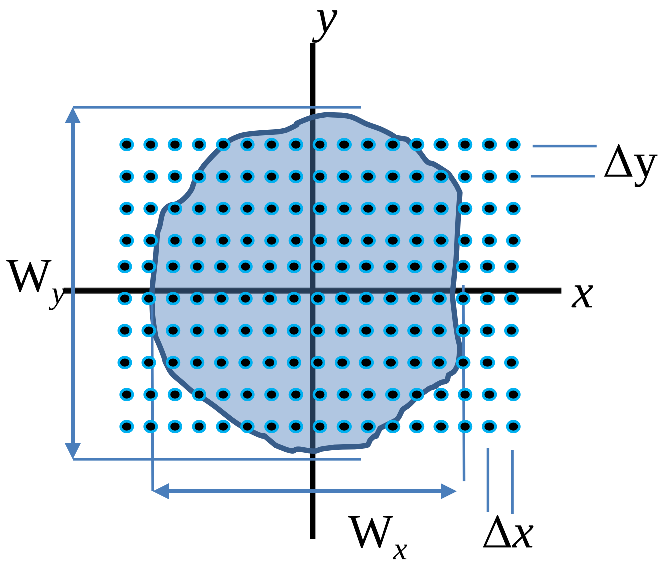


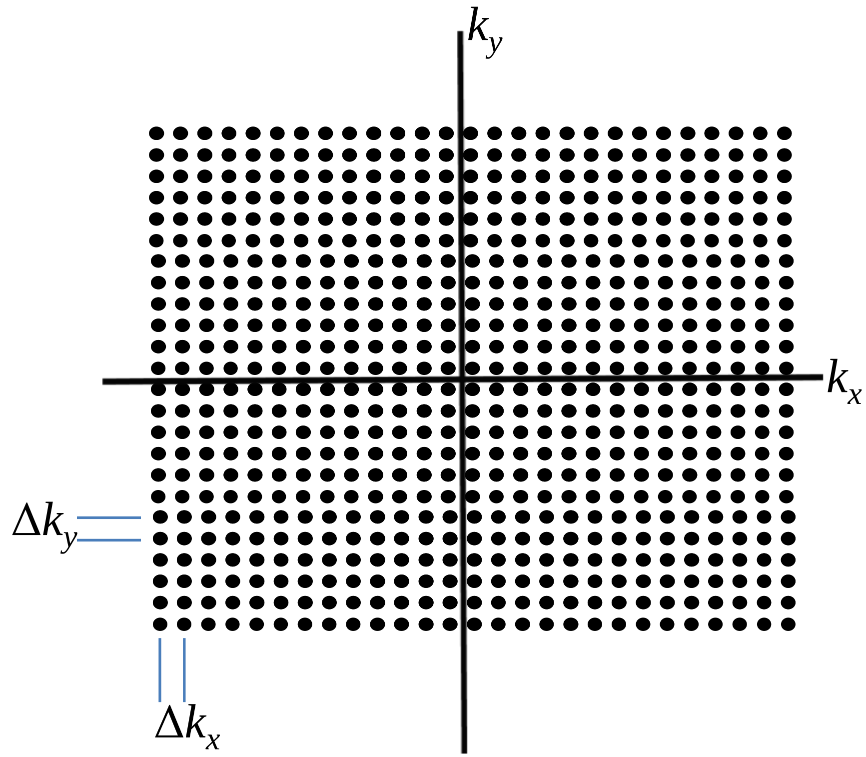


$$\Delta k_x = \left(\frac{\gamma}{2\pi}\right) G_x \Delta t \qquad \Delta k_y = \left(\frac{\gamma}{2\pi}\right) \Delta G_y t_y$$

$$\Delta k_x \leq \frac{1}{W_x}, \Delta k_y \leq \frac{1}{W_y}$$

Image resolution and sampling…



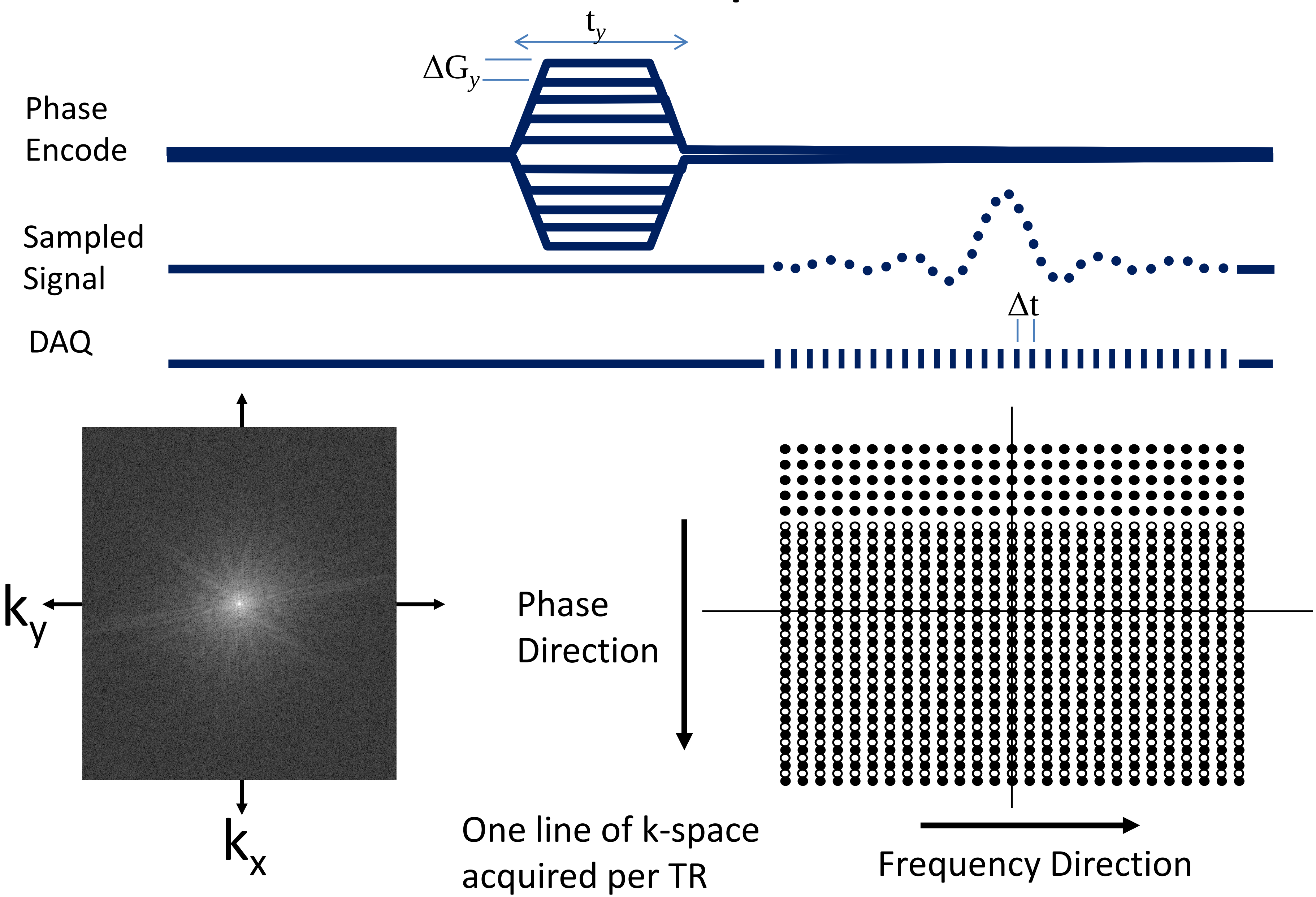


$$\Delta x = \frac{W_x}{N}, \Delta y = \frac{W_y}{N}$$

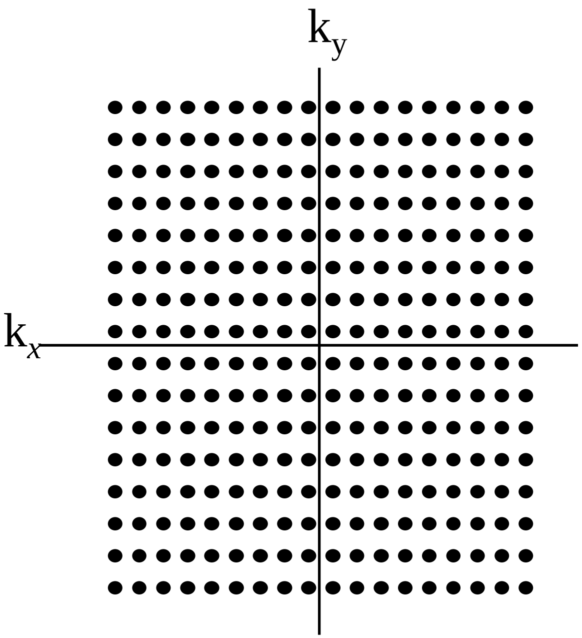
$$\Delta k_x \le \frac{1}{W_x}, \Delta k_y \le \frac{1}{W_y}$$

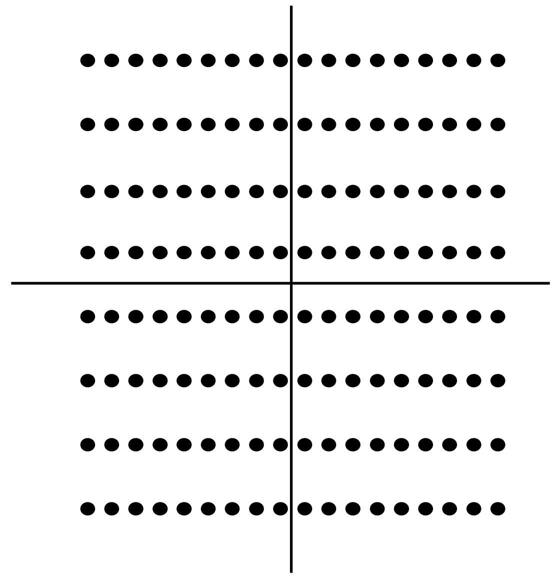
$$\Delta x = \frac{1}{N\Delta k_x}, \Delta y = \frac{1}{N\Delta k_y}$$

Rectilinear acquisition

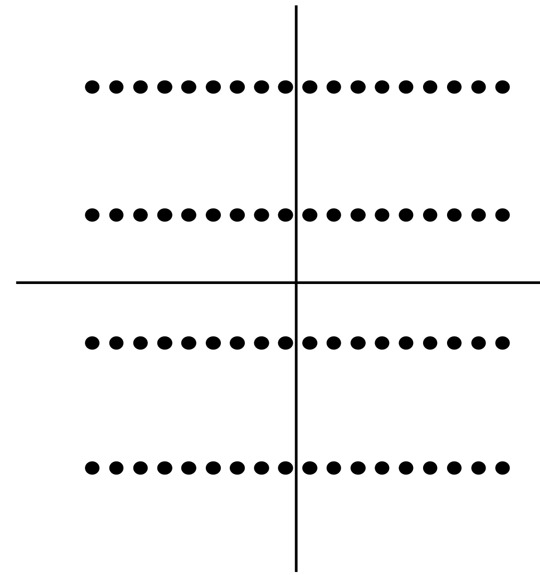


Undersampling (Rectilinear Acquisition)

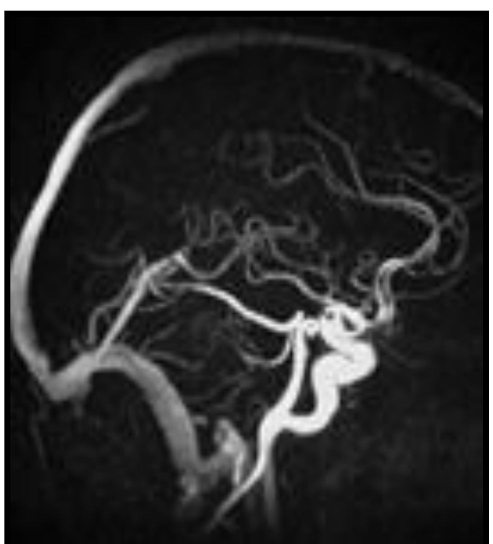


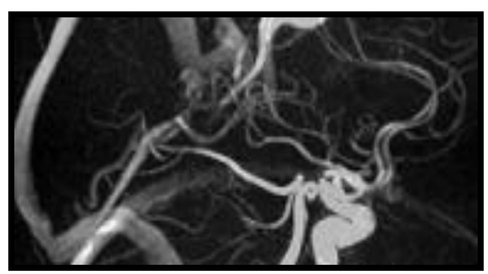


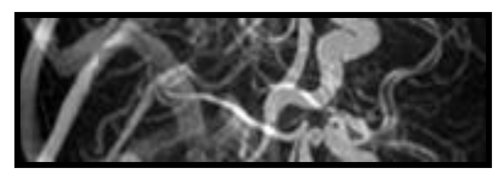
½ encodes



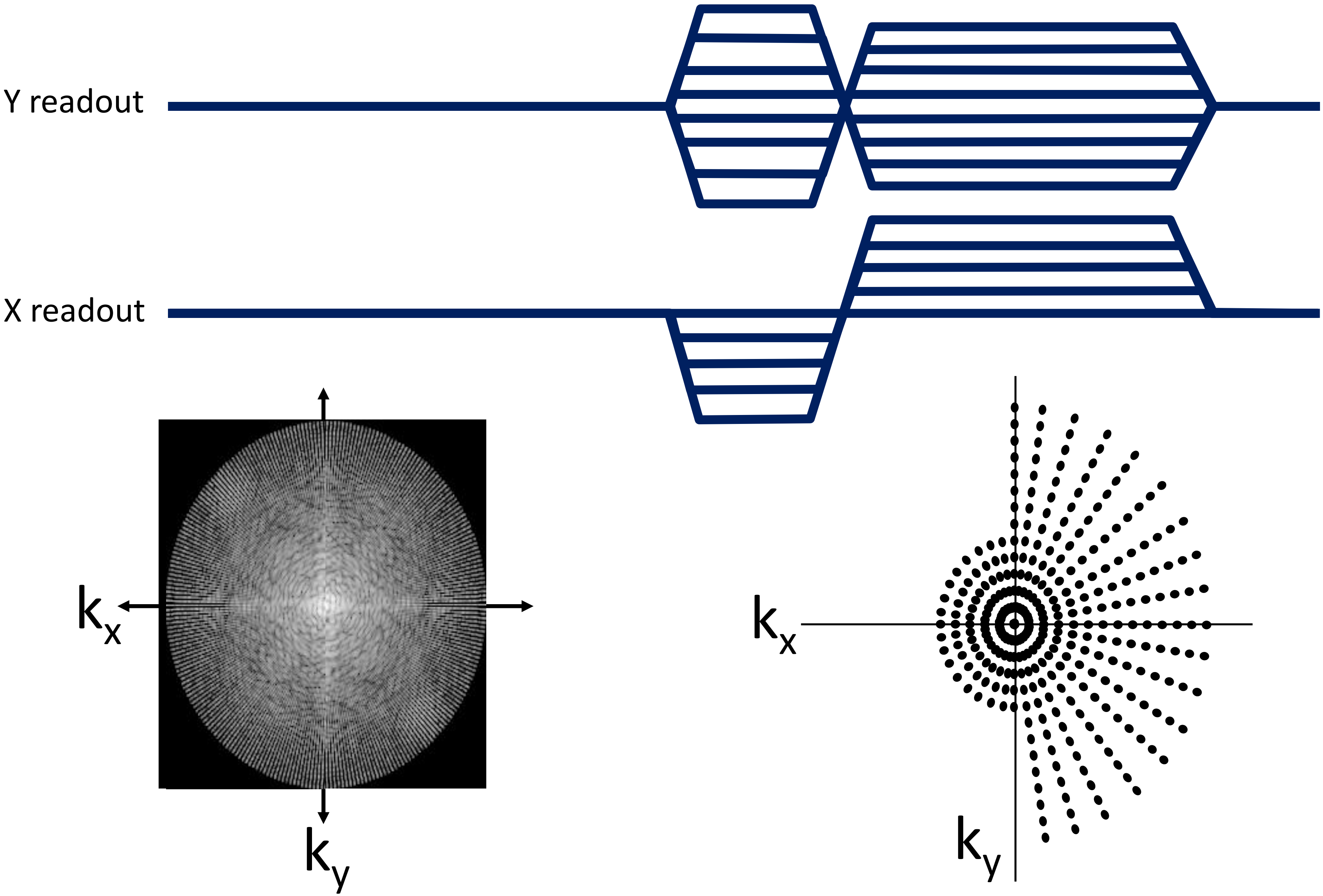
¼ encodes



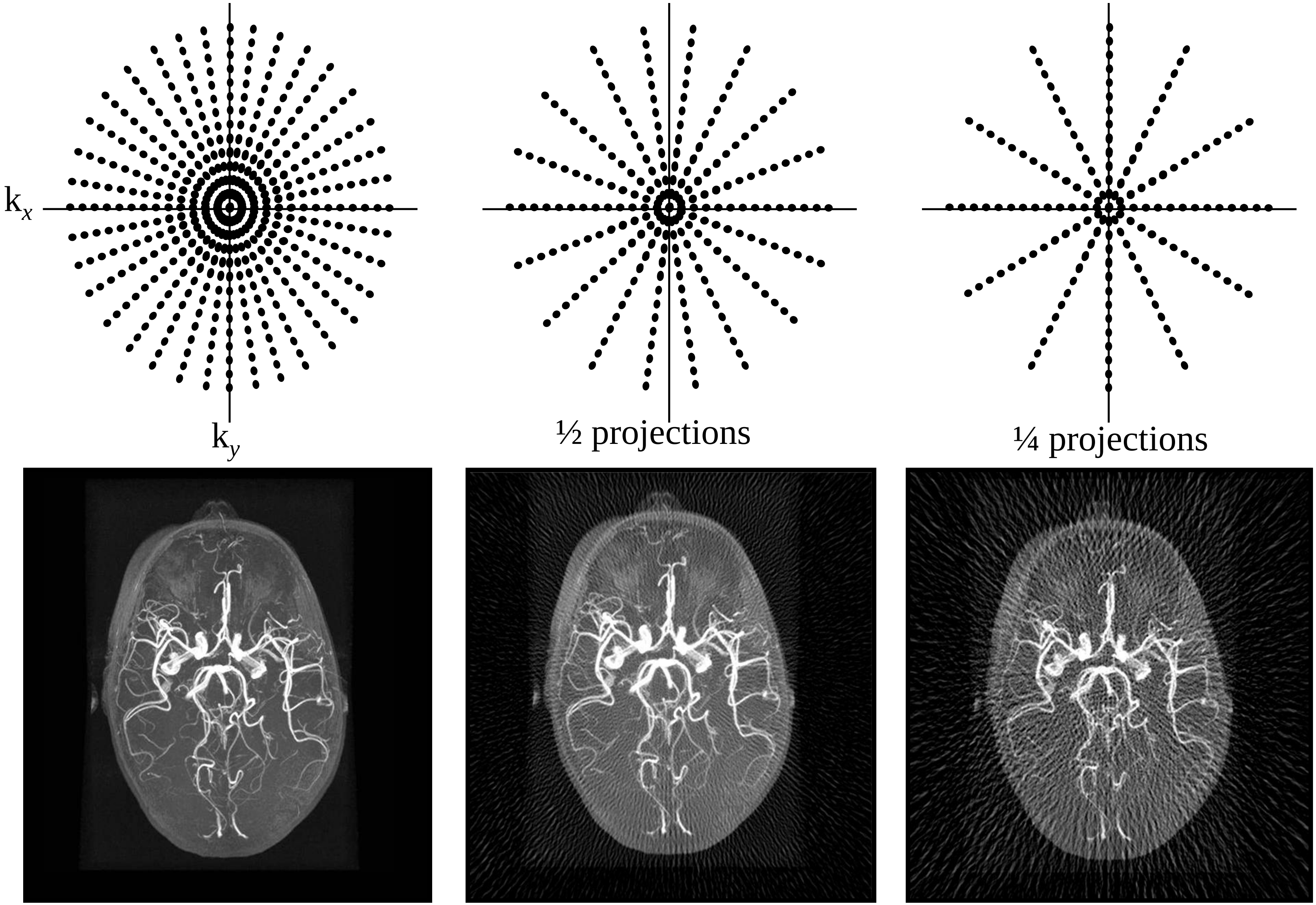




k-Space Acquisition (Radial Sampling)



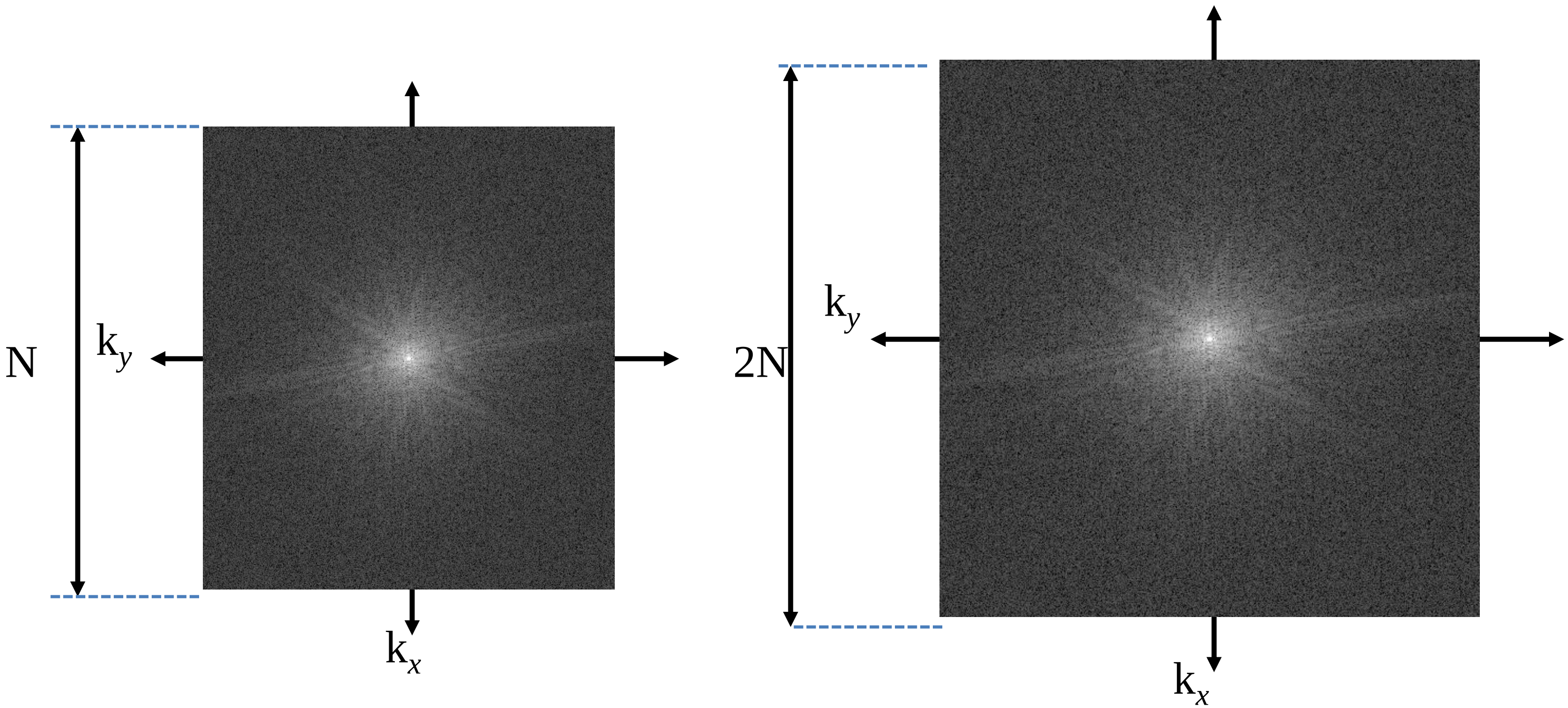
Accelerated 2D Radial Sampling



Trade-offs

- Image Resolution
- Signal to Noise ratio (SNR)
- Acquisition Time

Resolution Vs. Acquisition time



Double the resolution -> Double the time

SNR-When resolution is fixed

- Consider an Impulse object centered at origin
  - -Constant signal amplitude 'A' in k-space
  - -Each sample contains independent noise with variance σ^2

$$SNR = \frac{\sum_{j=1}^{N} A}{\sqrt{\sum_{j=1}^{N} \sigma_N^2}} = \frac{NA}{\sqrt{N\sigma_N^2}} = \frac{A\sqrt{N}}{\sigma_N}$$

SNR obtained
By simply adding samples

- The effect of signal averaging-SNR improvement

$$SNR = \frac{\sum_{j=1}^{N} 2A}{\sqrt{\sum_{j=1}^{N} 2\sigma_N^2}} = \frac{2NA}{\sqrt{2N\sigma_N^2}} = \frac{A\sqrt{2N}}{\sigma_N}$$

$$SNR \propto \sqrt{N_{avg}}$$

SNR-Accelerated scan-fixed resolution

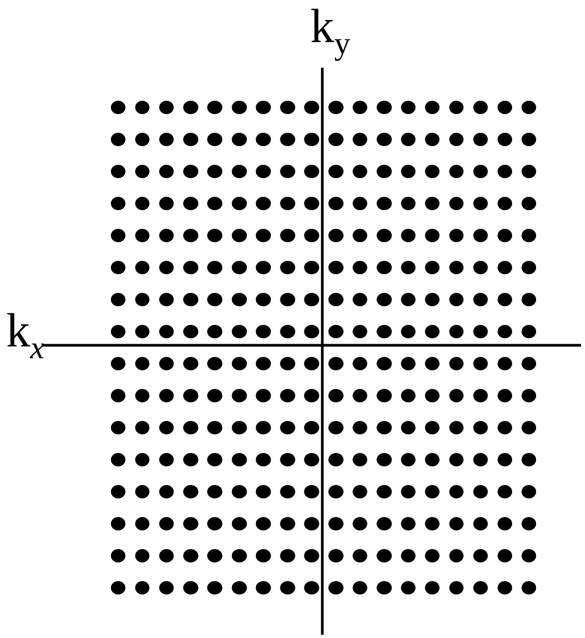
- Acceleration factor of 2-collect N/2 samples

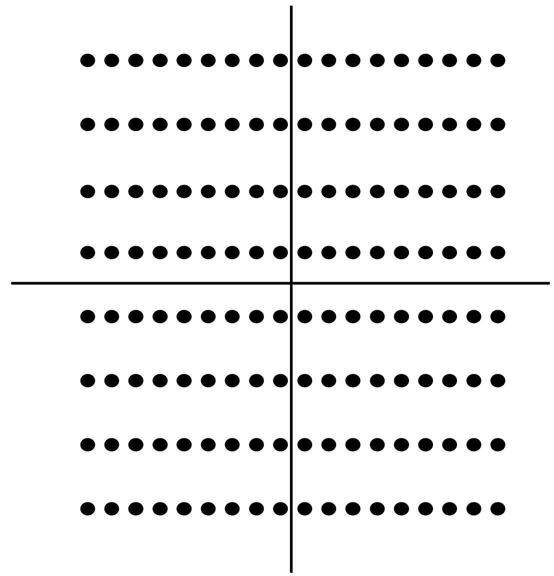
$$SNR = \frac{\sum_{j=1}^{N} A}{\sqrt{\sum_{j=1}^{N} \sigma_N^2}} = \frac{\frac{N}{2} A}{\sqrt{\frac{N}{2} \sigma_N^2}} = \frac{A}{\sigma_N} \sqrt{\frac{N}{2}}$$

- In general- In an accelerated scan with image resolution fixed

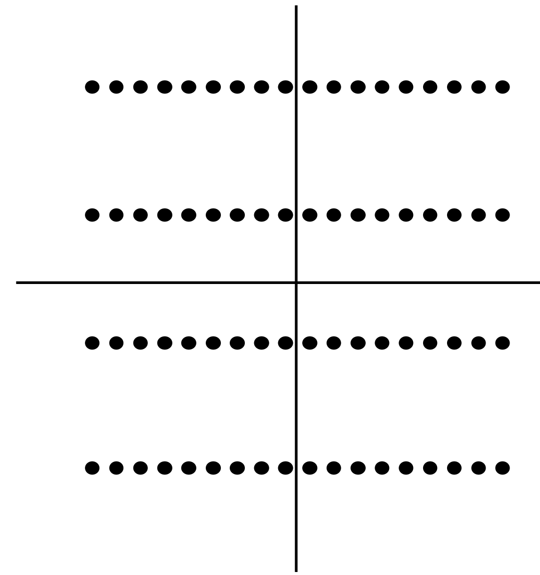
$$SNR \propto \frac{1}{\sqrt{R}}$$

Undersampling (Rectilinear Acquisition)

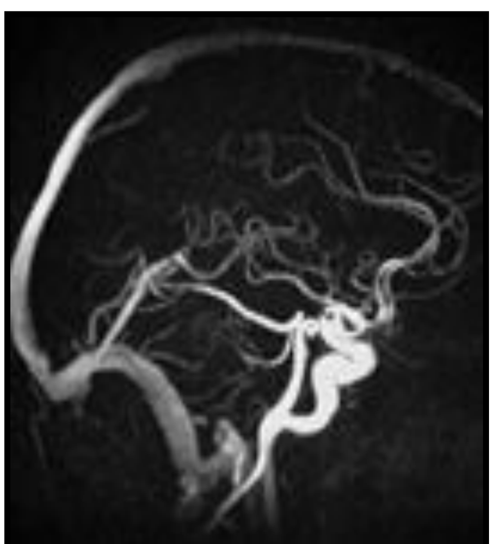


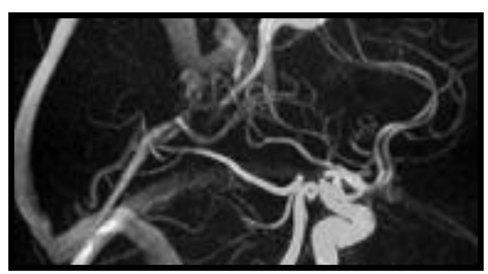


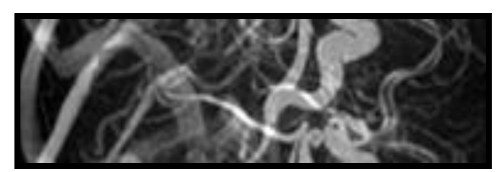
½ encodes – 30% lower SNR



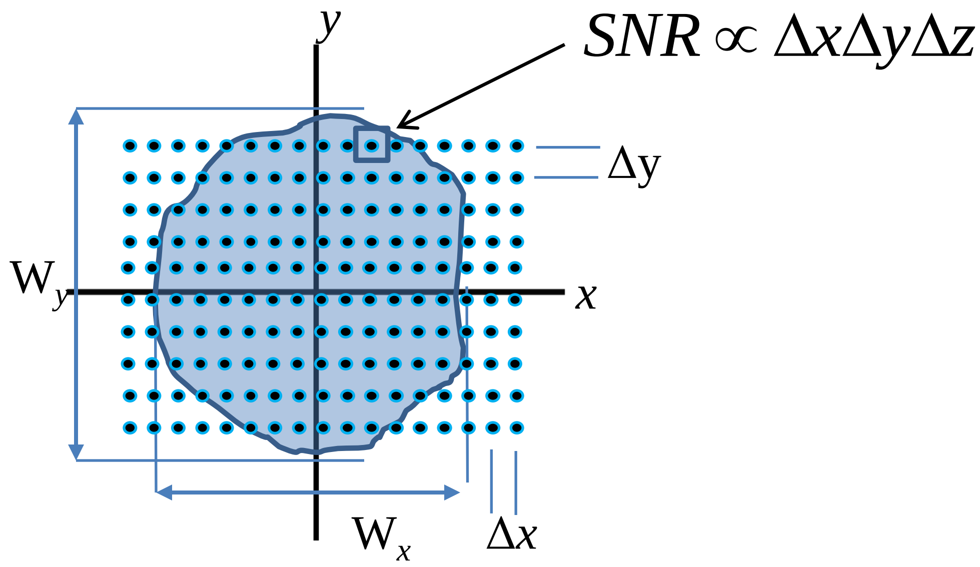
¼ encodes -50% lower SNR







SNR -When Resolution is not fixed



$$SNR \propto \Delta x \Delta y \Delta z \sqrt{\frac{N}{R}} F\left(M(x,y,z), \text{T1}, \text{T2}\right)$$

SNR – Resolution *Vs* Acquired samples *Vs* Acceleration factor

To summarize..

Acquired data	↓	Resolution	↔	SNR	↓	Acceleration	↑
Acquired data	↑	Resolution	↔	SNR	↑	Acceleration	↓
Acquired data	↑	Resolution	↑	SNR	↓	Acceleration	↓
Acquired data	↓	Resolution	↑	SNR	↓↓	Acceleration	↑
Acquired data	↓	Resolution	↓	SNR	↑	Acceleration	↑

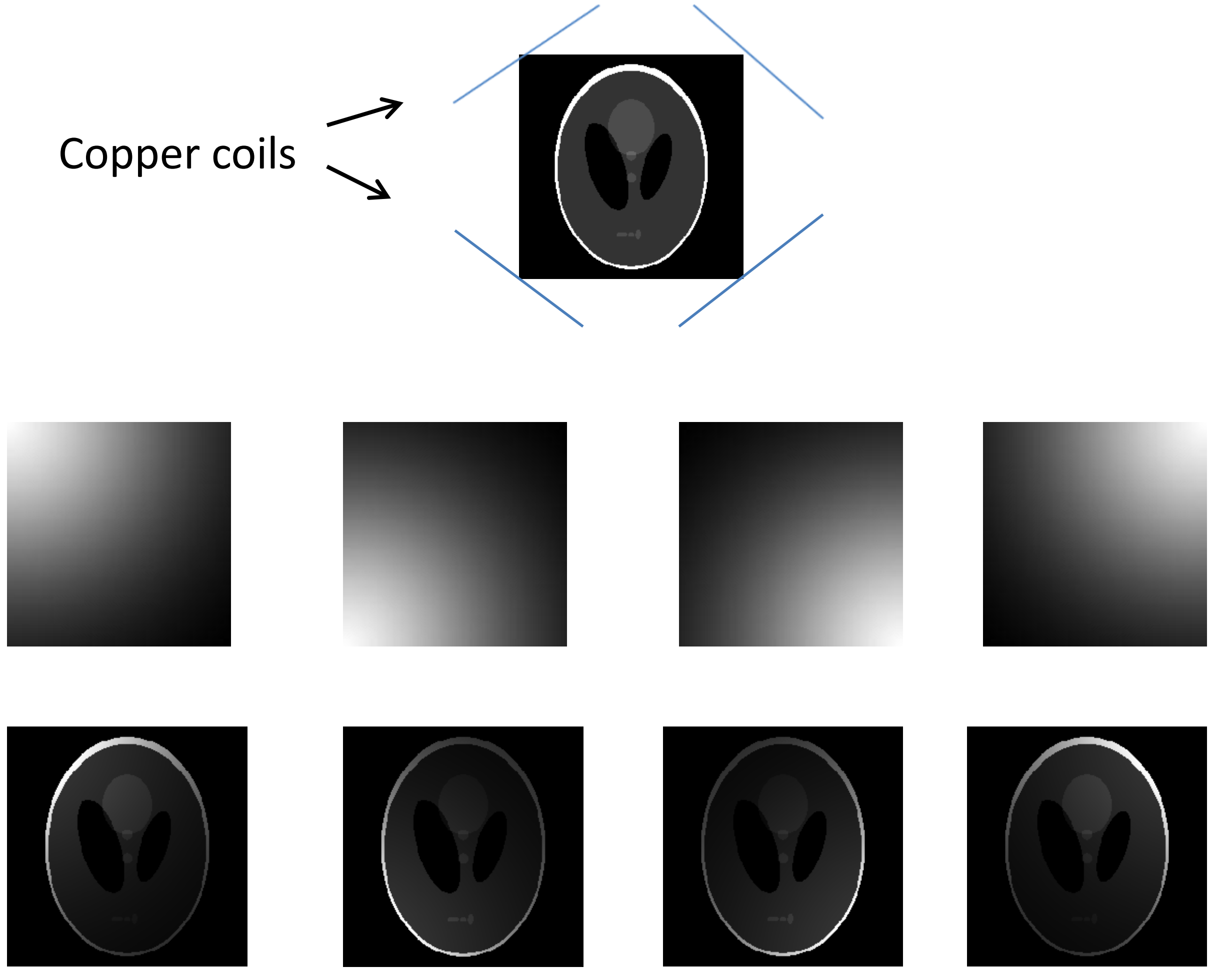
- Increasing intrinsic SNR can allow for greater acceleration factor

Parallel Imaging, Compressed Sensing and Aliased k-space acquisitions

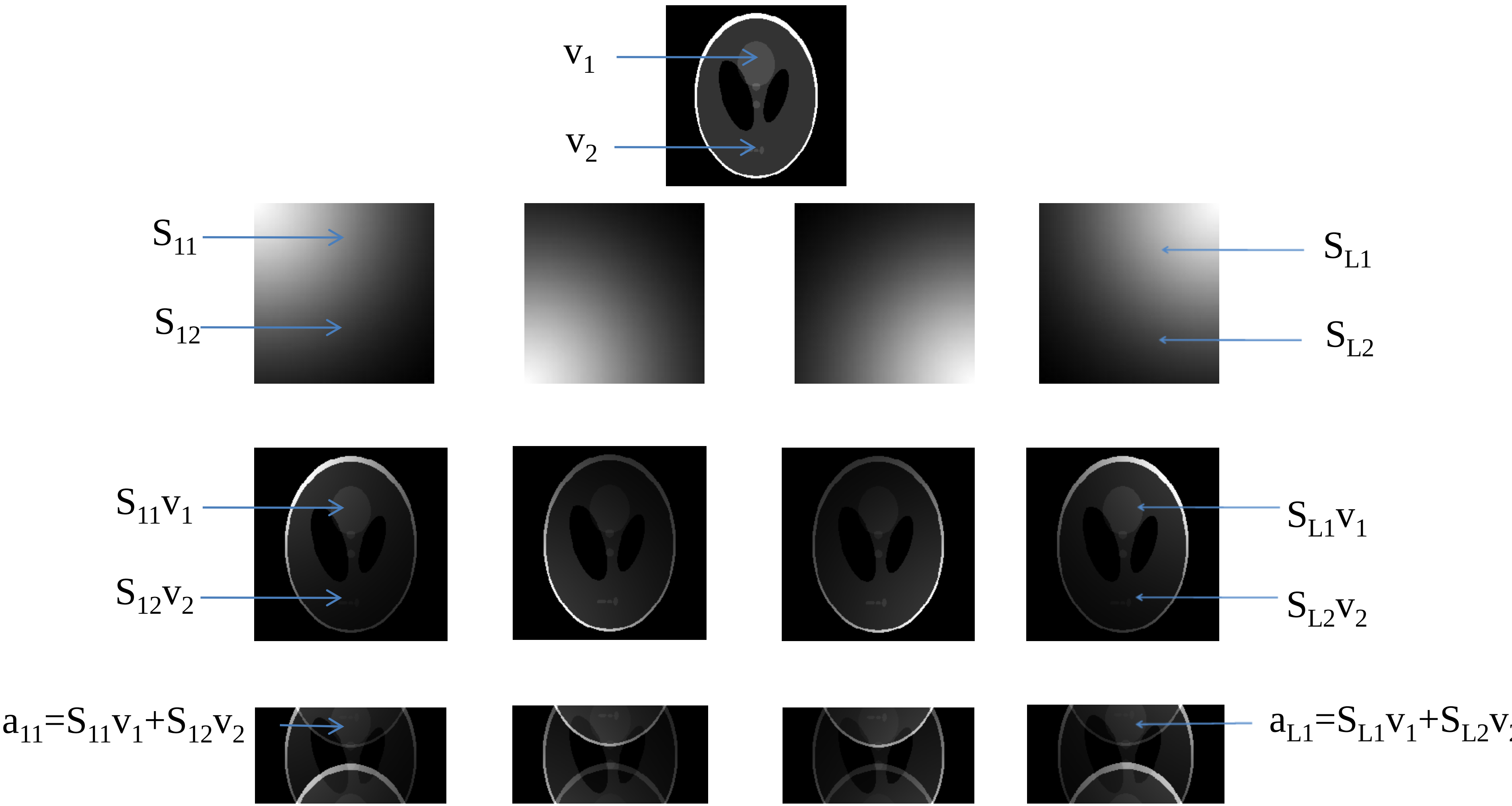
Parallel Imaging

Acquired data	↓	Resolution	↔	SNR	↓	Acceleration	↑
Acquired data	↓	Resolution	↑	SNR	↓↓	Acceleration	↑

Impact of coil sensitivity profiles



Impact of coil sensitivity profiles



Parallel Imaging-SENSE[1]

$$S_{(\gamma,\rho)} = s_\gamma\left(r_\rho\right)$$

γ^{th} Receiver sensitivity value at the r_p location

$$U = \left(S^H S\right)^{-1} S^H$$

Matrix that can restore the original voxels

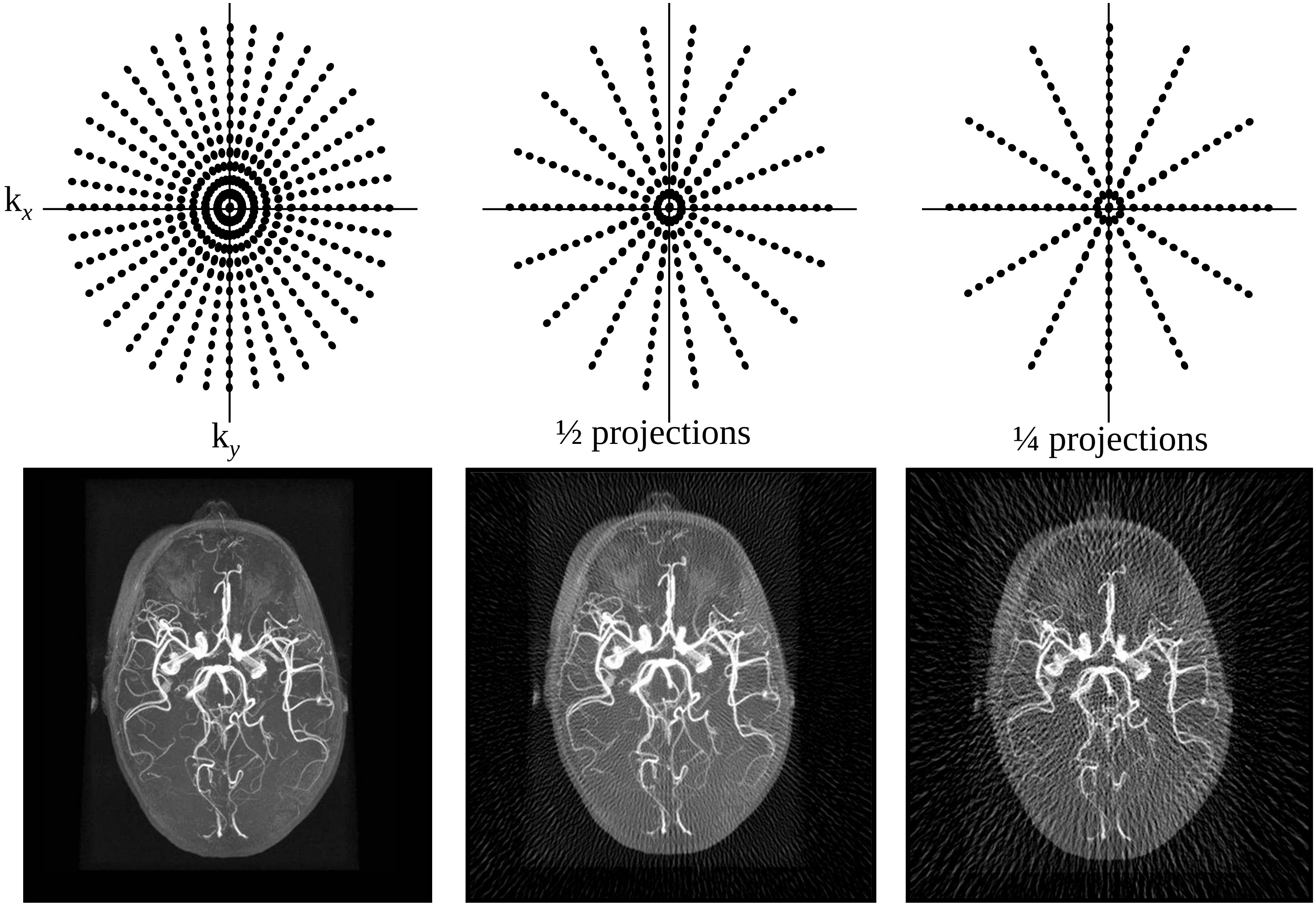
$$\mathrm{V} = \mathrm{Ua}$$

The equation to solve to restore original image

These equations work only for Rectilinear acquisitions!

[2]. *Pruessmann et al*. Magnetic Resonance in Medicine 42:952–962 (1999)

What about for Radial Sampling ?



SENSE in more general terms..

$$F(k_x, k_y) = \iint M(x, y) \overbrace{S_l(x, y) e^{-i2\pi k_y y} e^{-i2\pi k_x x}} dxdy$$

$$\downarrow$$

$$E_{(l,\rho)} = e^{jk_k r_\rho} s_l(r_\rho)$$

$$\downarrow$$

$$Ev = m$$

m - Undersampled k-space data

$$\downarrow$$

$$v = (E^h E) E^h m$$

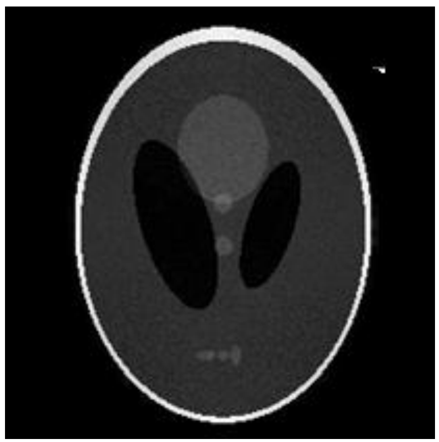
v – Unaliased image

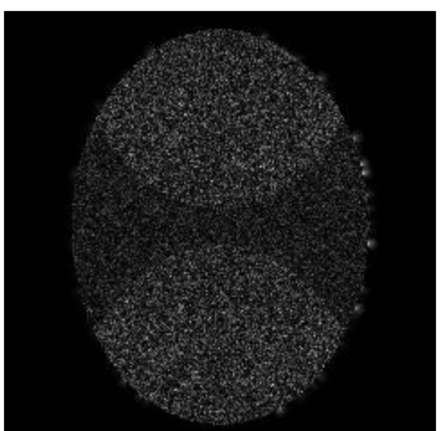
Noise propagation in SENSE

- SNR loss during acquisition $SNR \propto \frac{1}{\sqrt{R}}$

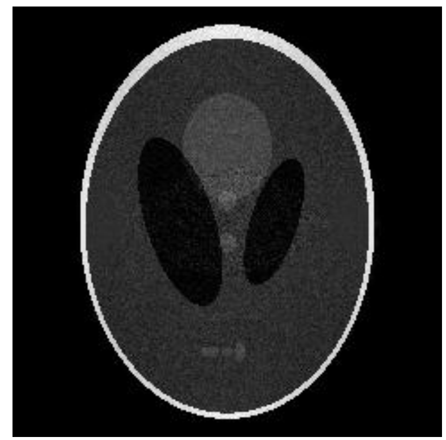
During reconstruction…

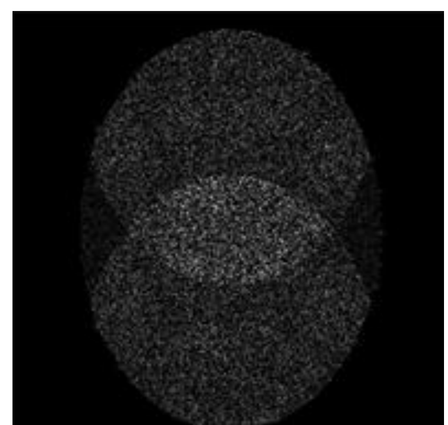
R=2





R=3



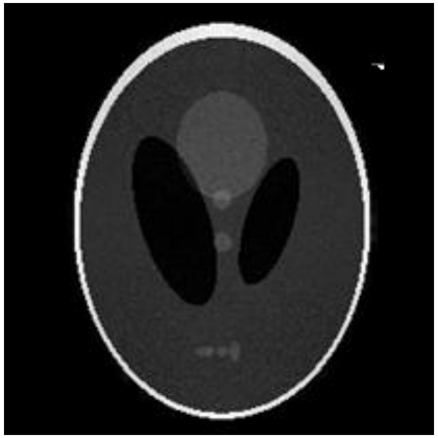


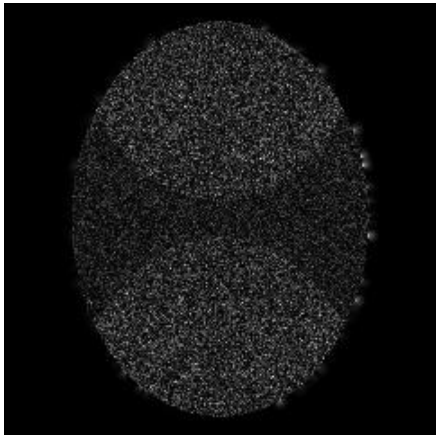
Noise propagation in SENSE

- SNR loss during acquisition $SNR \propto \frac{1}{\sqrt{R}}$

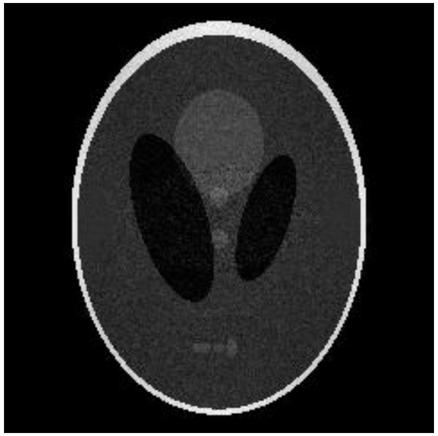
During reconstruction…

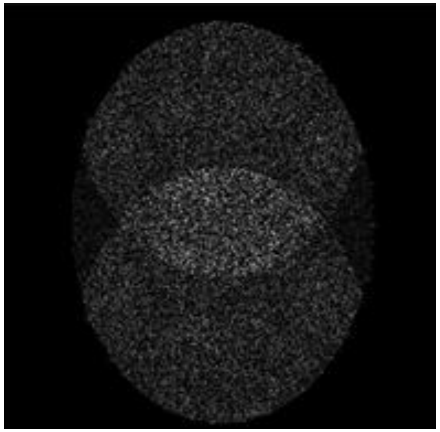
R=2





R=3



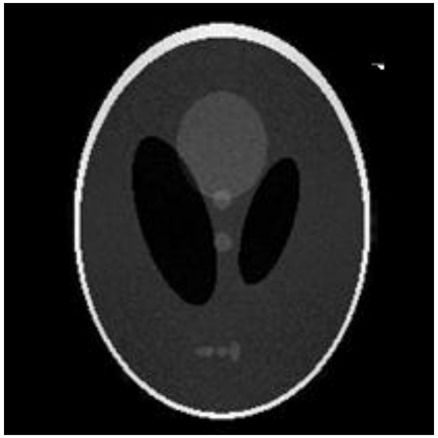


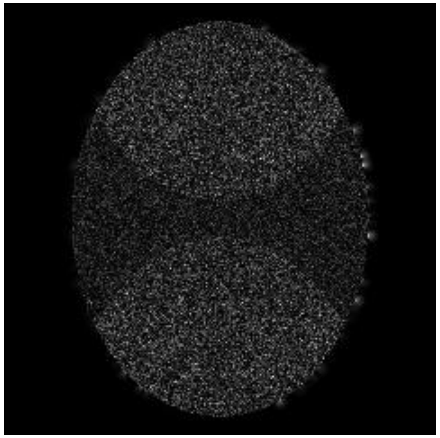
Noise propagation in SENSE

- SNR loss during acquisition $SNR \propto \frac{1}{\sqrt{R}}$

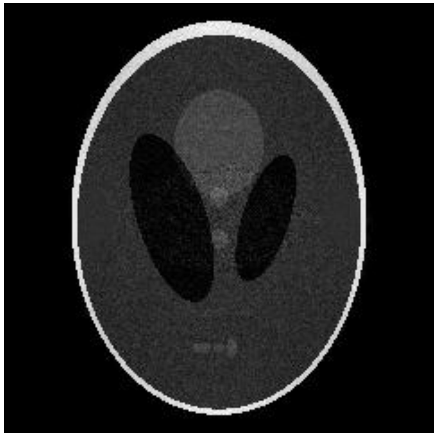
During rectilinear data reconstruction…

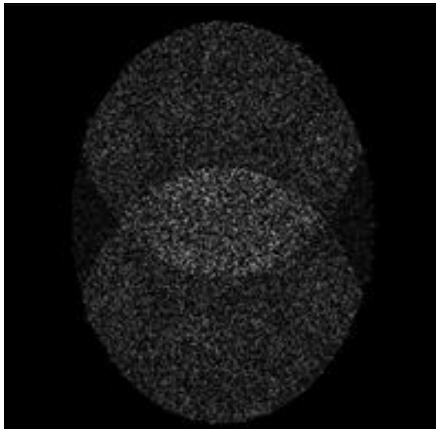
R=2





R=3





The noise propagation model

Noise in the γ^{th} channel: $n_\gamma(t) = \sum_\tau \alpha_{\gamma,\tau} \xi_\tau(t)$

Modeled as weighted sum of individual noise sources

Noise variance in the ρ^{th} voxel in rectilinear acquisition:

$$\sigma_\rho^2 = \sum_\tau \sigma_\tau^2 \left| \sum_{l,k} U_{\rho,(l,k)} \alpha_{l,\tau} \right|^2 \quad \text{and} \quad U = \left(S^H S\right)^{-1} S^H$$

Rearranging the variance equation, we get the following noise matrix (*X*):

$$X = U \psi U^H \quad \text{where} \quad \psi_{l,l'} = \sum_\tau \sigma_\tau^2 \alpha_{l,\tau} \alpha_{l,\tau}^*$$

Substitute *U* to get the following: $X = \left(S^H \psi^{-1} S\right)^{-1}$

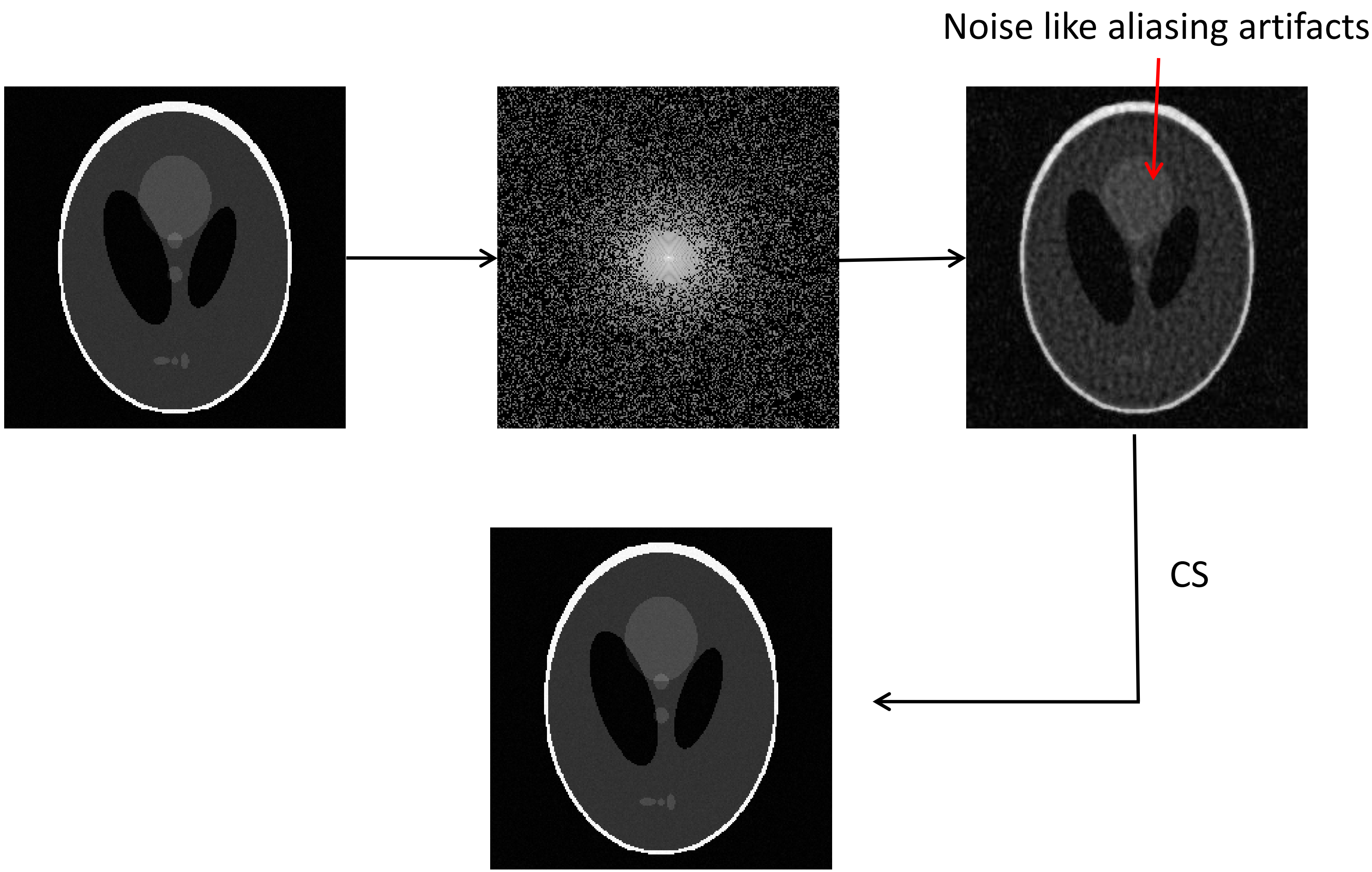
Limitations of Parallel Imaging

- Guaranteed SNR loss during acquisition and reconstruction
- Limits on the maximum acceleration
- Demanding Clinical applications need more acceleration

Compressive Sensing

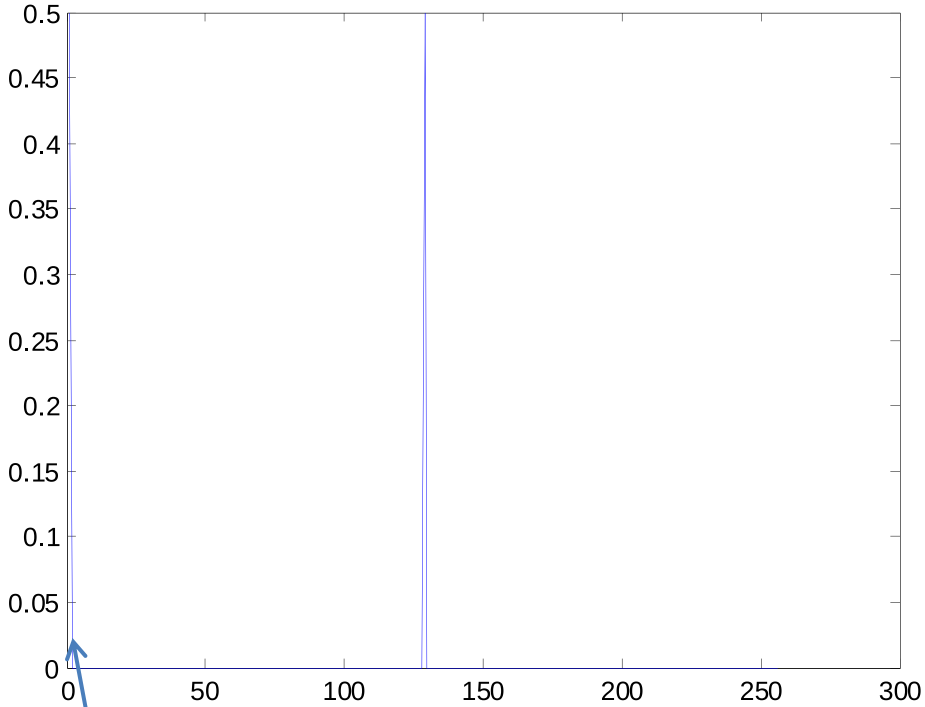
Acquired data	↓	Resolution	↔	SNR	↓	Acceleration	↑
Acquired data	↓	Resolution	↑	SNR	↓↓	Acceleration	↑

Compressed Sensing example

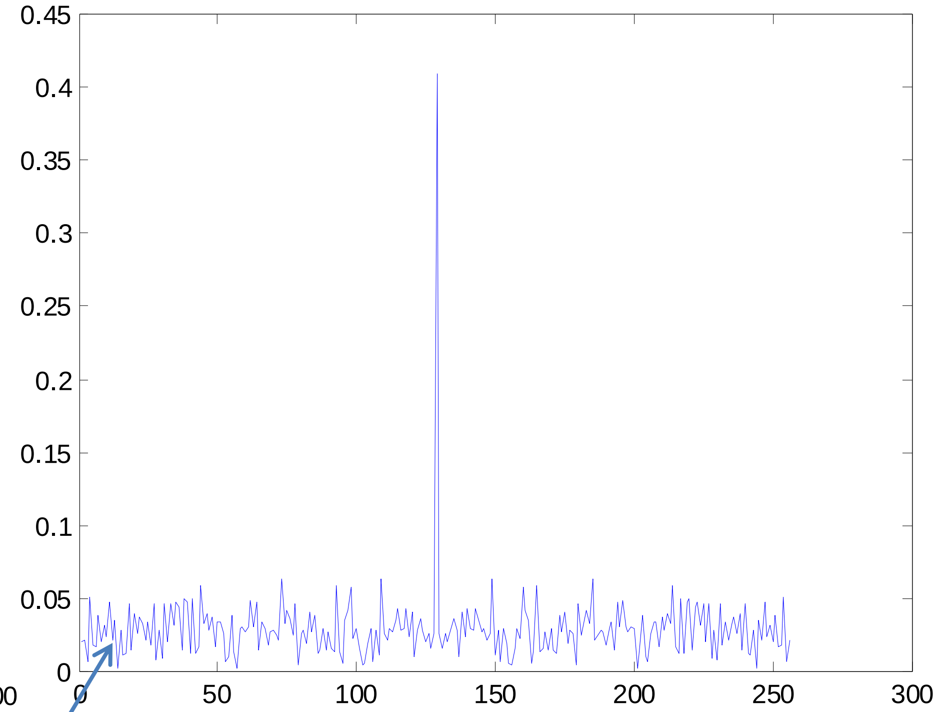


Idea behind random under-sampling?

1D Imaging example



Regular under-sampling factor of 2



Random under-sampling factor of 2

Regular aliasing artifacts

Noise like aliasing artifacts

CS reconstruction framework

Minimize $\left\| \boldsymbol{\psi} v \right\|_{l1}$ Ψ-Sparse representation

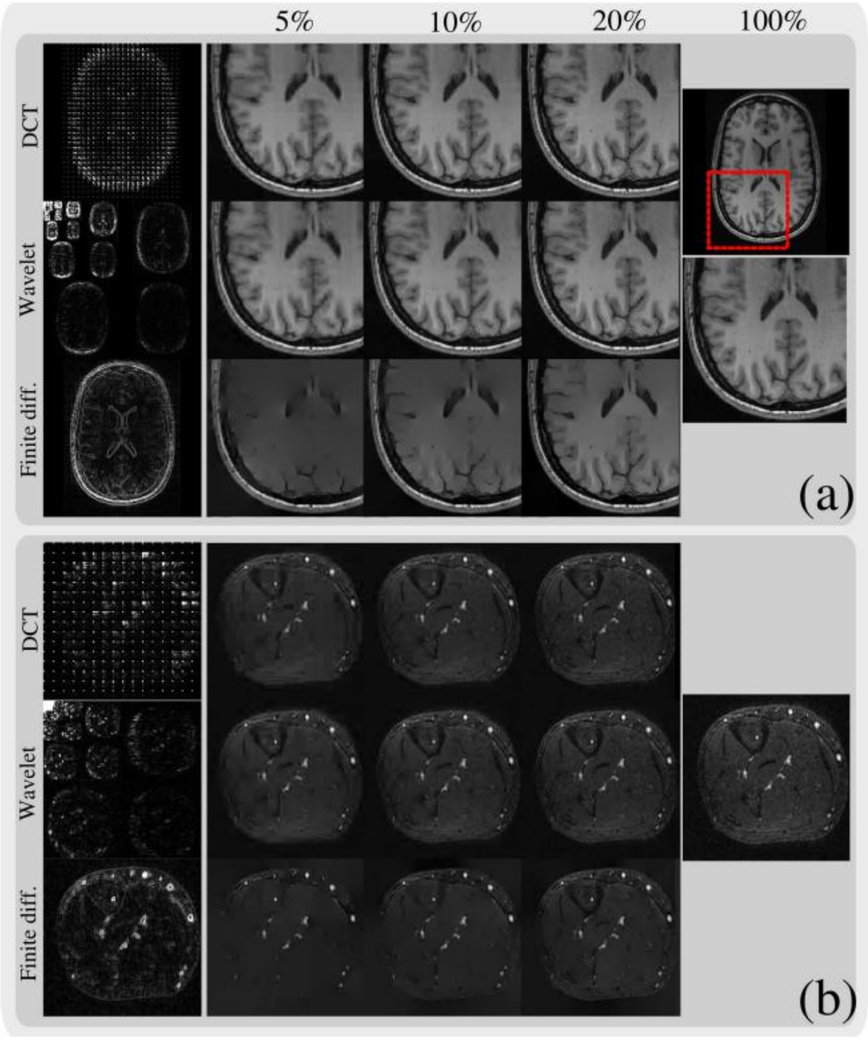
s.t $\left\| Ev - m \right\|_{l2} \leq \varepsilon$ Data consistency

In MRI…TV transformation is always used..

Minimize $\left\| \boldsymbol{\psi} v \right\|_{l1} + \eta TV\left(v\right)$ TV-Total variation

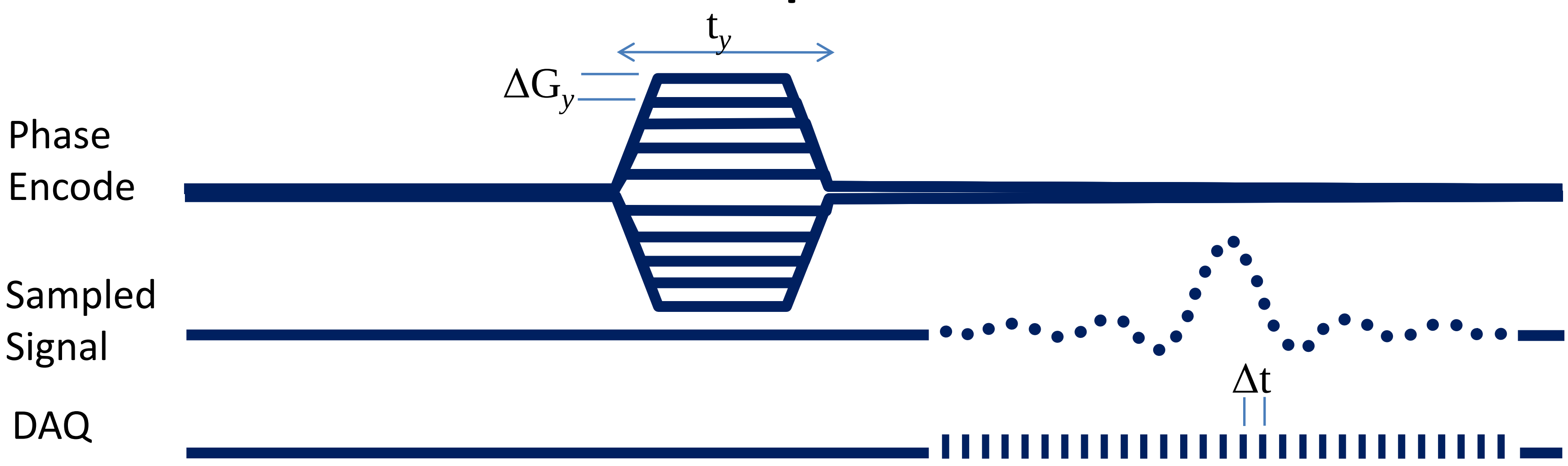
s.t $\left\| Ev - m \right\|_{l2} \leq \varepsilon$ Data consistency

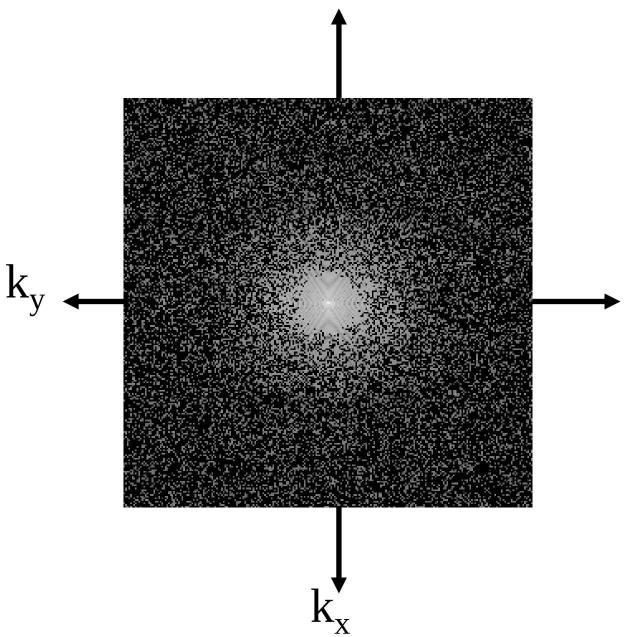
Sparse representation of MRI Images[3]

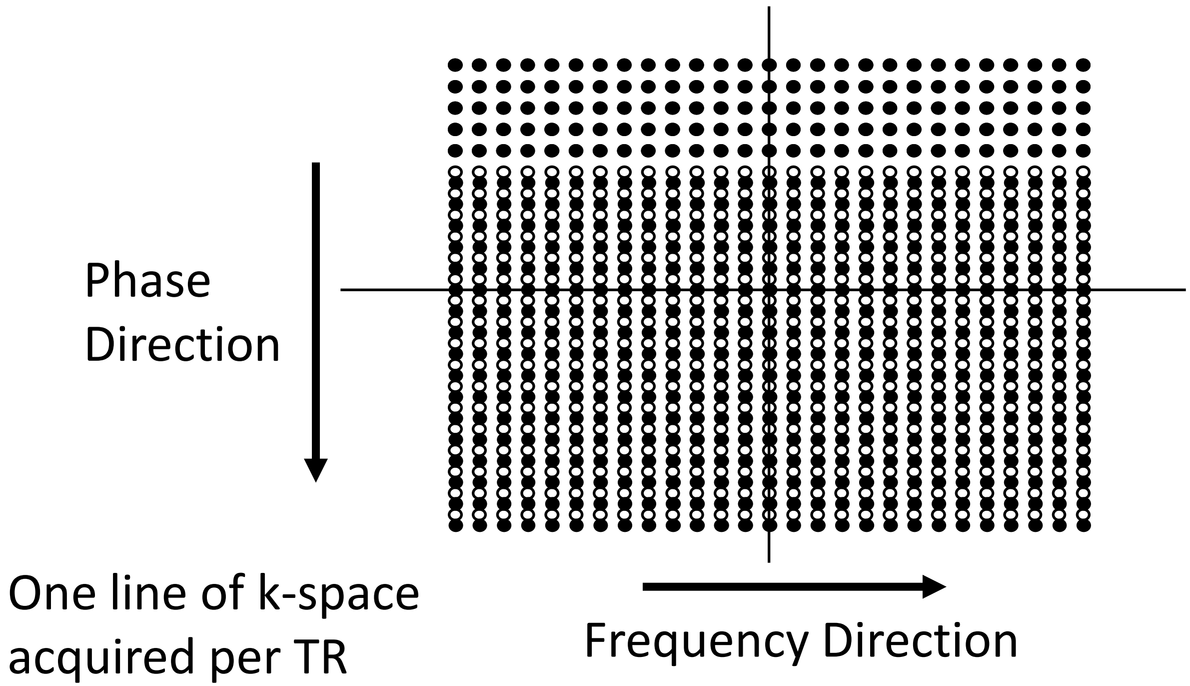


[3]. *Lustig et al*. Magnetic Resonance in Medicine 58(6):1182–1195 (2007)

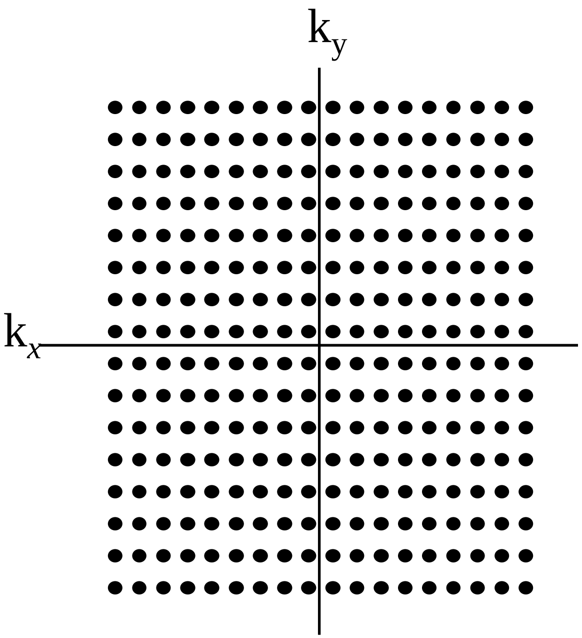
2D Rectilinear acquisition with CS?

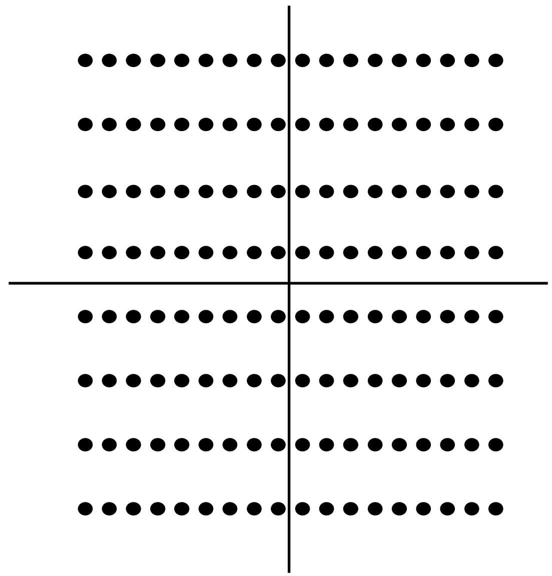




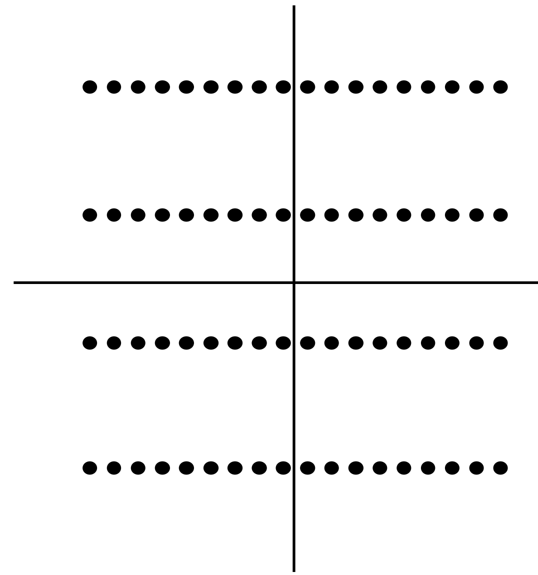


Conventional Rectilinear undersampling?

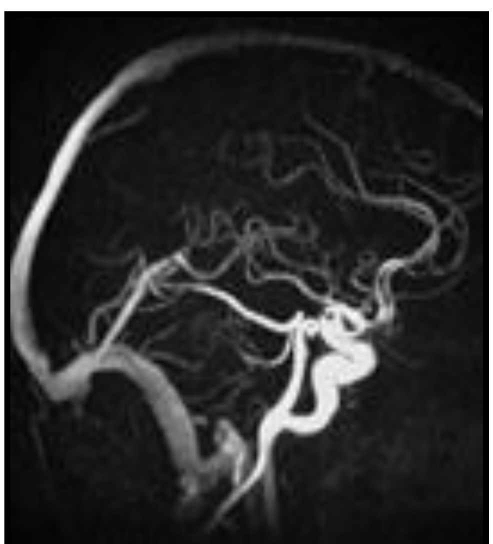


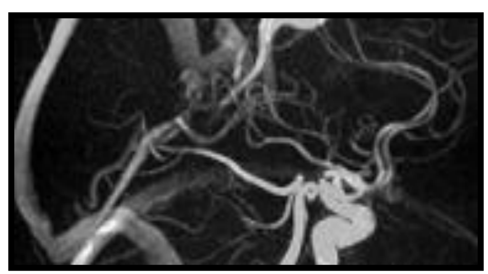


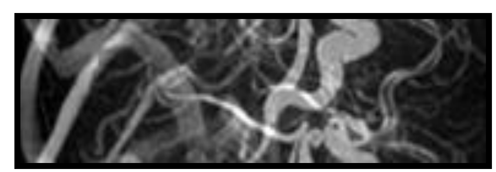
½ encodes



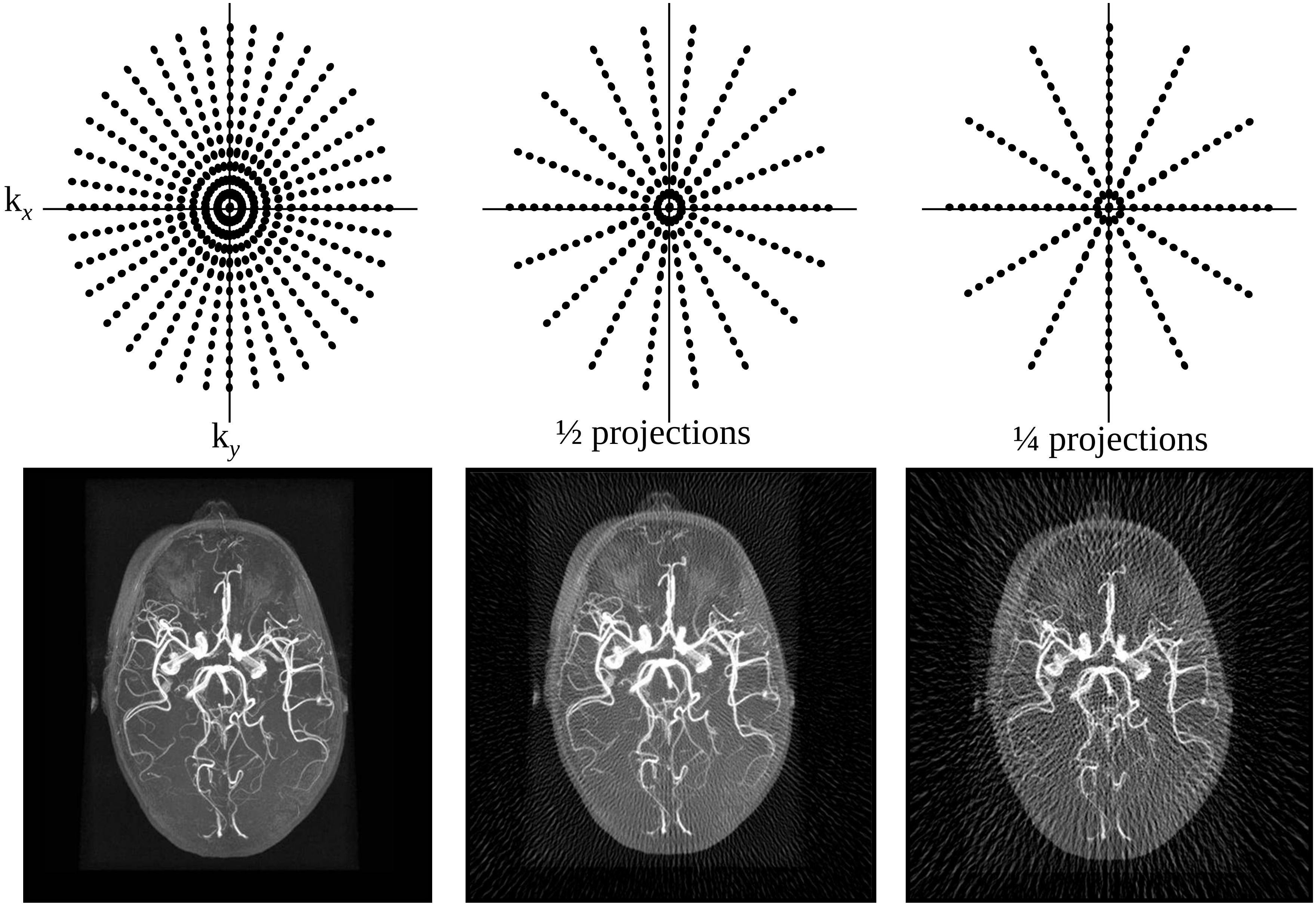
¼ encodes







What about for Radial Sampling ?



Limitations of Compressed Sensing

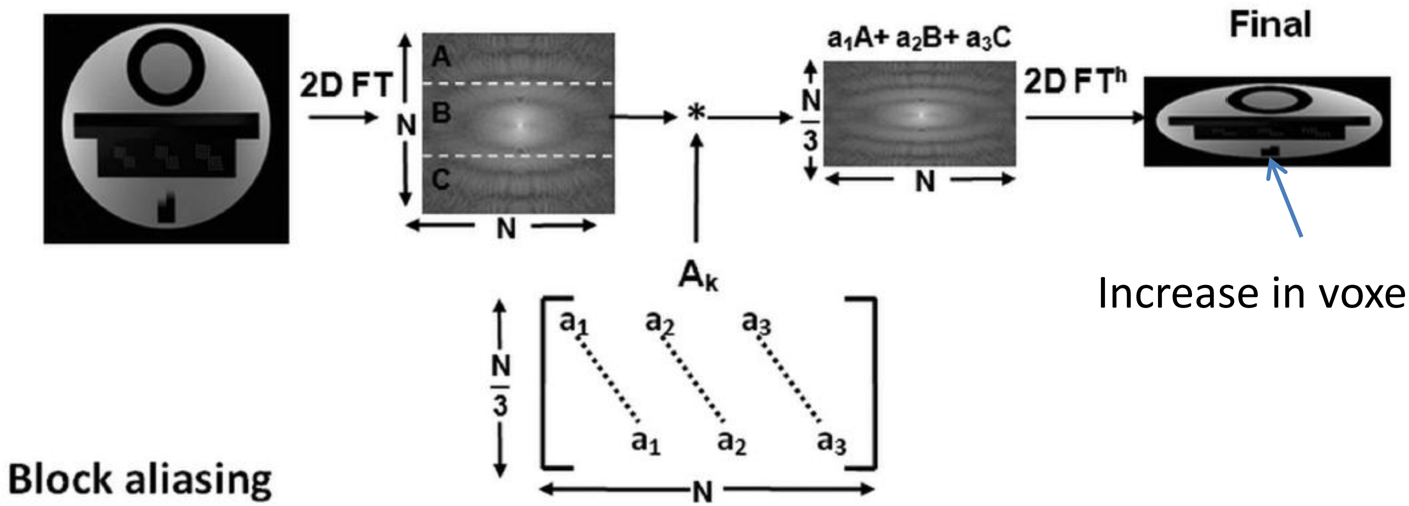
- Guaranteed SNR loss during acquisition
- Limits on the maximum acceleration
- Texture loss!
- Not applicable for all acquisitions (3D, Dynamic imaging most suitable)
- Compatibility with multi-receiver acquisitions not clear
- Visually improved or real restoration?

Aliased k-space acquisitions

Acquired data ↓	**Resolution** ↓	**SNR** ↑	**Acceleration** ↑

Can resolution be restored during reconstruction?

An example of overlapping k-space[4]



Increase in voxel size!

- All data acquired through overlapping
- Acceleration achieved due to parallel acquisition of k-space data
- Acquisition SNR increase due to voxel size increase

[4].Arunachalam . Early view. Magnetic Resonance in Medicine

Aliased k-space acquisitions

$$F(k_x,k_y) = \iint M(x,y) S_l(x,y) \overbrace{\left(a_1 e^{-i2\pi k_{y1} y} + a_2 e^{-i2\pi k_{y2} y} + a_3 e^{-i2\pi k_{y3} y}\right)} e^{-i2\pi k_x x} dxdy$$

$$\downarrow$$

$$E_{(l,\rho)} = \left(a_1 e^{jk_{k1} r_\rho} + a_{21} e^{jk_{k2} r_\rho} + a_3 e^{jk_{k3} r_\rho}\right) s_l\left(r_\rho\right)$$

$$\downarrow$$

$$Ev = m$$

m – Aliased k-space data

$$\downarrow$$

$$v = \left(E^h E\right) E^h m$$

v – Unaliased image

SNR gain in "aliased" acquisitions

- Consider an Impulse object centered at origin

$$SNR_{ref} = \frac{\sum_{j=1}^{N} A}{\sqrt{\sum_{j=1}^{N} \sigma_N^2}} = \frac{NA}{\sqrt{N\sigma_N^2}} = \frac{A\sqrt{N}}{\sigma_N}$$

SNR obtained
By simply adding samples

$$SNR_k = \frac{|A|\sqrt{a_1^2 + a_2^2 + a_3^2}\sqrt{N}\sqrt{3}}{\sigma_N}$$

$$\frac{SNR_k}{SNR_{ref}} = \sqrt{a_1^2 + a_2^2 + a_3^2}\sqrt{3}$$

Limitations of Aliased k-space acquisitions

- Poorly conditioned encoding matrix E
- Sensitive to coil geometry
- Required several receivers
- Increase TR duration

To summarize..

- Parallel Imaging in commercial scanners as a clinical imaging product
- Compressed Sensing – work in progress..
- Aliased k-space acquisitions – Undergoing clinical investigations..
- Use of receiver sensitivities proven technology
- Sparsity constraints unpredictable – Sampling patterns need development